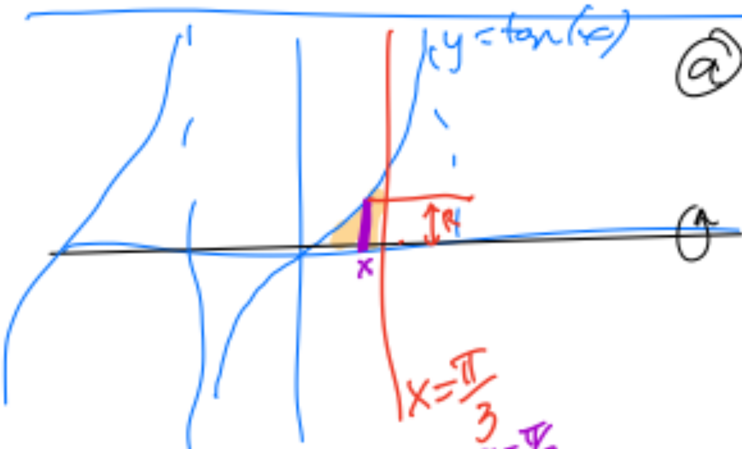
Example Let F be the region between $y = \tan x$, $x = \pi/3$ and the x-axis.

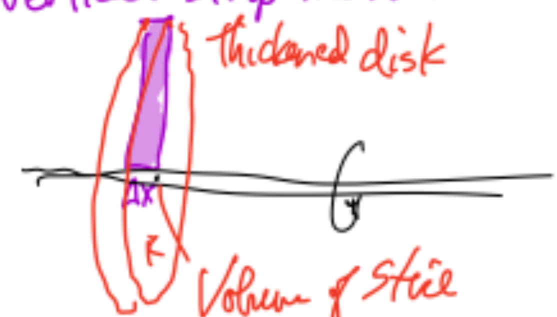
ⓐ Find the volume generated by revolving F around the x-axis. (2 ways)

ⓑ Find the volume generated by revolving F around $y = 2$ (2 ways)

ⓒ Find the volume generated by revolving F around $x = -\frac{\pi}{2}$. (2 ways)



ⓐ Vertical Strip method



Volume of slice $= \pi R^2 \Delta x$

ⓐ $\text{Volume} = \int_{x=0}^{x=\pi/3} (\text{volume of slice}) = \int_0^{\pi/3} \pi R^2 \, dx$

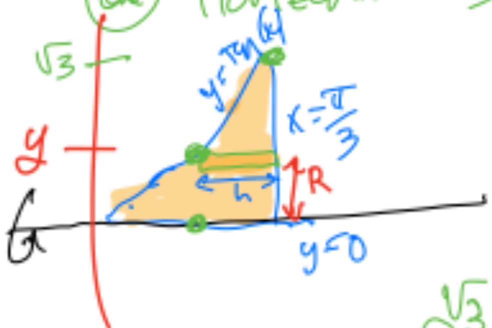
$= \int_0^{\pi/3} \pi \left(\tan(x) - 0\right)^2 dx = \pi \int_0^{\pi/3} \tan^2(x) \, dx$

$= \pi \int_0^{\pi/3} \underbrace{\sec^2(x)}_{-1} \, dx = \pi \left(\tan(x) - x\right)\Big|_0^{\pi/3}$

$= \pi\left(\tan\left(\frac{\pi}{3}\right) - \frac{\pi}{3}\right) - \pi(0-0)$

$= \boxed{\pi\left(\sqrt{3} - \frac{\pi}{3}\right)}$

(b) Horizontal Slice Method:



Volume of slice

$V = 2\pi R h \Delta y$

Cylindrical shell

$y = \tan(x)$
$x = \arctan(y)$

$$\text{Volume} = \int_0^{\sqrt{3}} 2\pi R h \, dy$$

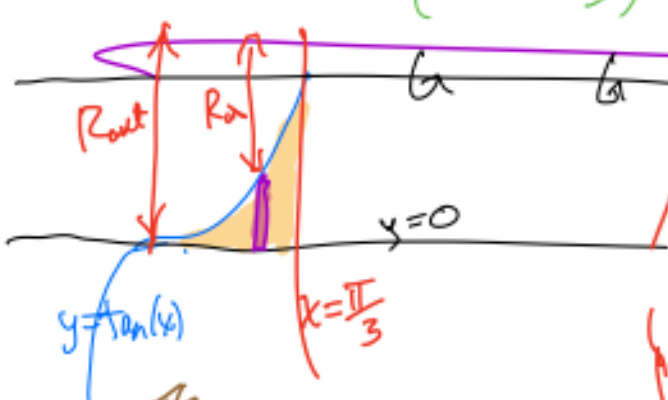
$R = y - 0 = \tan(x) - 0$

$h = \frac{\pi}{3} - x$
$= \frac{\pi}{3} - \arctan(y)$

$$= \int_0^{\sqrt{3}} 2\pi(y)\left(\frac{\pi}{3} - \arctan(y)\right) dy$$

$=$ (parts $u = \frac{\pi}{3} - \arctan(y)$, $dv = 2\pi y \, dy$)

$= \pi\left(\sqrt{3} - \frac{\pi}{3}\right)$



— Volume of slice

$\left(\pi(R_{out}^2) - \pi(R_{in}^2)\right)\Delta x$

$R_{out} = 2 - 0$

$R_{in} = y_{top} - y_{bottom}$
$= 2 - \tan(x)$

$$\text{Volume} = \int_0^{\pi/3} \left(\pi R_{out}^2 - \pi R_{in}^2\right) dx$$

$$= \int_0^{\pi/3} \left(\pi(2^2) - \pi(2-\tan(x))^2\right) dx = \dots -$$

Same volume with horiz strips.

Volume of slice - cylindrical shell

$2\pi R h \Delta y$

$R = y_{top} - y_{bottom} = 2 - y$

$h = x_{big} - x_{small}$

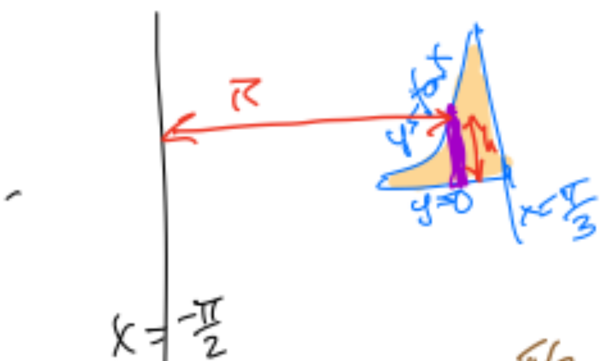
$$\text{Volume} = \int (\text{Volume of Slice}) = \int_0^{\sqrt{3}} 2\pi R h \, dy$$

$= \frac{\pi}{3} - x$

$= \frac{\pi}{3} - \arctan(y)$

$$= \int_0^{\sqrt{3}} 2\pi(2-y)\left(\frac{\pi}{3} - \arctan(y)\right) dy = \dots$$

(c) Revolve around $x = -\frac{\pi}{2}$



Volume of slice

$= \underline{2\pi R h \Delta x}$

$$\text{Volume} = \int_{x=0}^{\pi/3} 2\pi R h \, dx$$

$R = x_{big} - x_{small}$

$R = x - \left(-\frac{\pi}{2}\right)$

$R = x + \frac{\pi}{2}$

$h = y_{top} - y_{bot}$

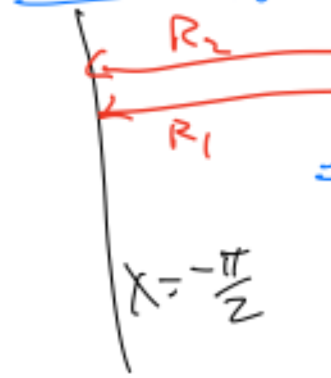
$= \tan(x) - 0$

$$= \int_0^{\pi/3} 2\pi\left(x + \frac{\pi}{2}\right)(\tan(x)) \, dx$$

parts $u = \left(x + \frac{\pi}{2}\right)$
$dv = \tan(x) \, dx$

$\dots = $

Method 2 horiz



$$\left(\pi R_2^2 - \pi R_1^2\right)\Delta y$$

$$R_2 = \frac{\pi}{3} - \frac{-\pi}{2} = \frac{5\pi}{6}$$

$$R_1 = x - \frac{-\pi}{2}$$

$$= \arctan(y) + \frac{\pi}{2}$$

$$\int_0^{\sqrt{3}} \left(\pi R_2^2 - \pi R_1^2\right) dy$$

$$= \int_0^{\sqrt{3}} \left(\pi\left(\frac{5\pi}{6}\right)^2 - \pi\left(\arctan(y) + \frac{\pi}{2}\right)^2\right) dy$$

$=$ parts $u = \left(\arctan(y) + \frac{\pi}{2}\right)^2$

$dv = dy$ . . . .

$=$